Atoms

Bohr's atomic model

V Vijaykiran
M.Sc., B.Ed., NET
SIGMA Physics Resource Centre
http://www.sigmaprc.in

Atoms

Bohr joined Rutherford for research in England. He recognized the flaws in Rutherford model and made corrections based on *postulates*.

Bohr's postulates

- An electron in an atom could revolve in certain stable orbits without the emission of radiant energy.

  According to this postulate, each atom has certain definite stable states in which it can exist, and each possible state has definite total energy. These are called the stationary states of the atom.

- Electron revolves around the nucleus only in those orbits for which the angular momentum is an integral multiple of $h/2\pi$.

  This postulate defines condition for stable orbits on the basis of quantization of angular momentum.

- An electron might make a transition from one non-radiating orbit to another of lower energy emitting a photon having energy equal to the energy difference between the initial and final states.

$$\boxed{h\nu = E_f - E_i}$$

This postulate justifies the emission of radiation from the atom.

Consider an electron of mass m and charge e revolving around the nucleus of charge Ze. Let r be the radius of the circular path. Electrostatic force on the electron is given by coulombs inverse square law

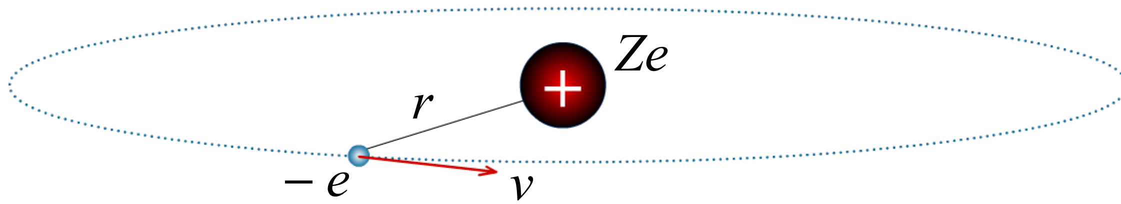
$$F = \frac{1}{4\pi\varepsilon_o}\frac{e\,Ze}{r^2} \qquad \text{— i}$$

Centripetal force on the electron is given by

$$F = \frac{mv^2}{r} \qquad \text{— ii}$$

Electrostatic force acts as the centripetal force therefore

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_o}\frac{e\,Ze}{r^2}$$



$$mv^2 = \frac{1}{4\pi\varepsilon_o}\frac{Ze^2}{r} \qquad \text{— iii}$$

Using Bohr's postulate of quantization of angular momentum we get

$$mvr = n\frac{h}{2\pi} \qquad \text{— iv}$$

$$r = n\frac{h}{2\pi mv} \qquad \text{— v}$$

Substituting this in eq (iii)

$$mv^2 = \frac{1}{4\pi\varepsilon_o}\frac{Ze^2\,2\pi mv}{nh}$$

$$\boxed{v = \frac{1}{2\varepsilon_o}\frac{Ze^2}{nh}} \qquad \text{— vi}$$

Using v in eq (iv) we get

$$mr\frac{Ze^2}{2\varepsilon_o nh} = \frac{nh}{2\pi}$$

$$\boxed{r = \frac{n^2h^2\varepsilon_o}{\pi mZe^2}} \qquad \text{— vii}$$

Energy

Velocity of the electron and the radius of its circular orbit are given by

$$\boxed{v = \frac{1}{2\varepsilon_o}\frac{Ze^2}{nh}} \quad \text{vi}$$

$$\boxed{r = \frac{n^2h^2\varepsilon_o}{\pi m Z e^2}} \quad \text{vii}$$

Kinetic energy

$$KE = \frac{1}{2}mv^2$$

$$\boxed{KE = \frac{mZ^2e^4}{8\varepsilon_o^{\,2}n^2h^2}} \quad \text{viii}$$

Potential energy

$$PE = \frac{1}{4\pi\varepsilon_o}\frac{Q_1Q_2}{r}$$

$$\boxed{PE = -\frac{mZ^2e^4}{4\varepsilon_o^{\,2}n^2h^2}} \quad \text{ix}$$

Total energy

$$TE = KE + PE$$

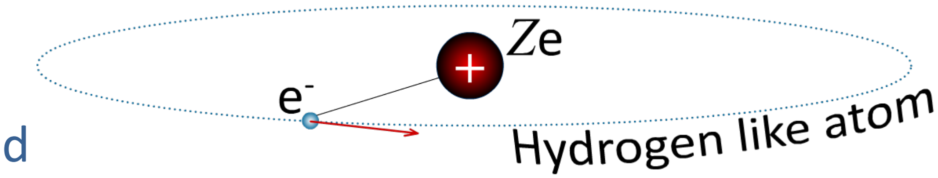
Using equations (viii) and (ix) we get

$$TE = \frac{mZ^2e^4}{8\varepsilon_o^{\,2}n^2h^2} - \frac{mZ^2e^4}{4\varepsilon_o^{\,2}n^2h^2}$$

$$\boxed{TE = -\frac{mZ^2e^4}{8\varepsilon_o^{\,2}n^2h^2}} \quad \text{x}$$

- Negative total energy implies bound state of the electron. Energy has to supplied to the electron to remove it from the atom
- Presence of n indicates that only certain energy levels are possible
- Presence of Z implies that the relation is applicable to *hydrogen like atoms*

Range of values in the atomic scale



$$v = \frac{1}{2\varepsilon_o}\frac{Ze^2}{nh}$$

Velocity in the ground state is 3.2 x 10^6 ms^{-1}

$$r = \frac{n^2h^2\varepsilon_o}{\pi m Ze^2}$$

$$\boxed{\frac{n^2}{Z}\times 0.529\ A}$$

r_o is 0.529 x 10^{-10} m

$$TE = -\frac{mZ^2e^4}{8\varepsilon_o^2n^2h^2}$$

$$\boxed{-\frac{Z^2}{n^2}13.6eV}$$

2.17 x 10^{-18} J

Some useful observations

- Angular momentum of an electron in n^{th} Bohr orbit is $nh/2\pi$.
- Torque acting on the electron is zero
- TE is –ve of KE
- PE is 2 x TE

Quantization of angular momentum , de-Broglie wavelength and stationary orbits

Bohr's postulate for quantization of angular momentum is

$$mvr = n\frac{h}{2\pi} \quad \text{—— i}$$

Using the de-Broglie relation, wavelength of matter wave associated with a particle of mass m moving with speed v is given by

$$\lambda = \frac{h}{mv} \quad \text{—— ii}$$

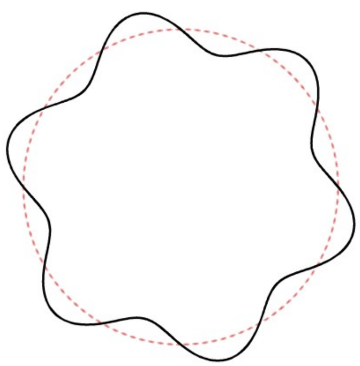
$$\Rightarrow \quad mv = \frac{h}{\lambda}$$

Substituting this in equation (i) we get

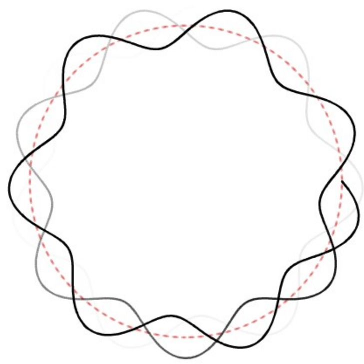
$$\frac{h}{\lambda}r = n\frac{h}{2\pi}$$

$$\boxed{2\pi r = n\lambda}$$

Above relation implies that the condition for ***stationary orbits*** is that the circumference of orbit should be an integral multiple of de-Broglie wavelength of the orbiting electron



Stationary orbit

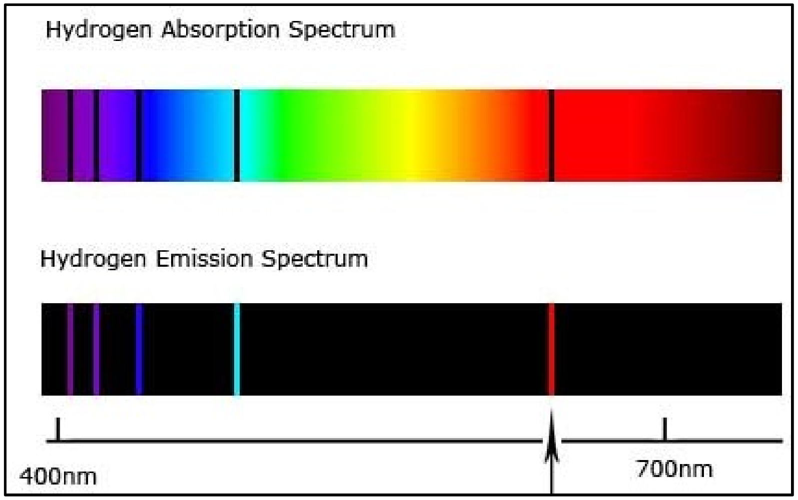


Atomic Spectra

When an atomic gas or vapour is excited at low pressure, usually by passing an electric current through it, the emitted radiation has a spectrum which contains certain specific wavelengths only. This is called *emission line spectrum*.

It consists of bright lines on a dark background.

Line spectra of a material can serves as a type of *fingerprint* for unique identification of the gas.



When white light passes through a gas and we analyze the transmitted light using a spectrometer we find some dark lines in the spectrum. These dark lines correspond precisely to those wavelengths which were found in the emission line spectrum of the gas.

This is called the *absorption spectrum* of the material of the gas.

Wavelength of radiation emitted in electronic transition

Total energy of the system is given by

$$TE = -\frac{mZ^2e^4}{8\varepsilon_o^{\,2}n^2h^2}$$

When an electron jumps from a higher orbit (n_i) to a lower orbit (n_f), change in the total energy is

$$\Delta E = -\frac{mZ^2e^4}{8\varepsilon_o^{\,2}h^2}\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right)$$

Energy of the photon emitted in such a transition is given by

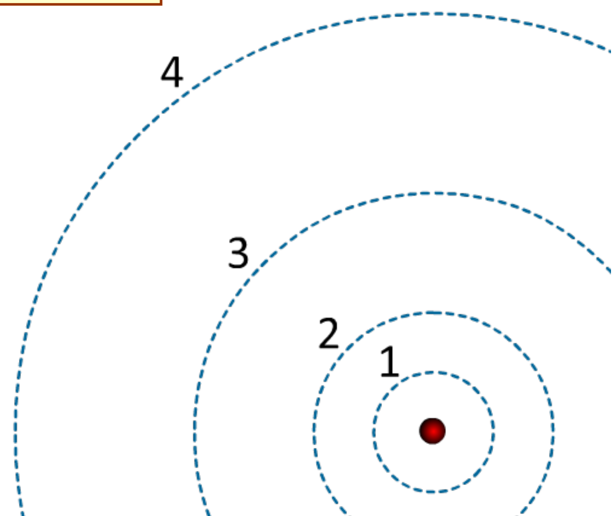
$$h\nu = \frac{mZ^2e^4}{8\varepsilon_o^{\,2}h^2}\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right)$$

$$\frac{hc}{\lambda} = \frac{mZ^2e^4}{8\varepsilon_o^{\,2}h^2}\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right)$$

$$\frac{1}{\lambda} = \frac{mZ^2e^4}{8\varepsilon_o^{\,2}h^3c}\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right)$$

Denoting $\dfrac{me^4}{8\varepsilon_o^{\,2}h^3c}$ as R i.e. Rydberg's constant ($1.097 \times 10^7\ m^{-1}$)

$$\boxed{\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right)}$$



$$\boxed{\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

Lyman series

$n_f = 1$ and $n_i = 2, 3, 4, \ldots \infty$

λ_{max} corresponds to smallest energy change hence $n_i = 2$

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\frac{1}{\lambda_{max}} = R\left(1 - \frac{1}{4}\right)$$

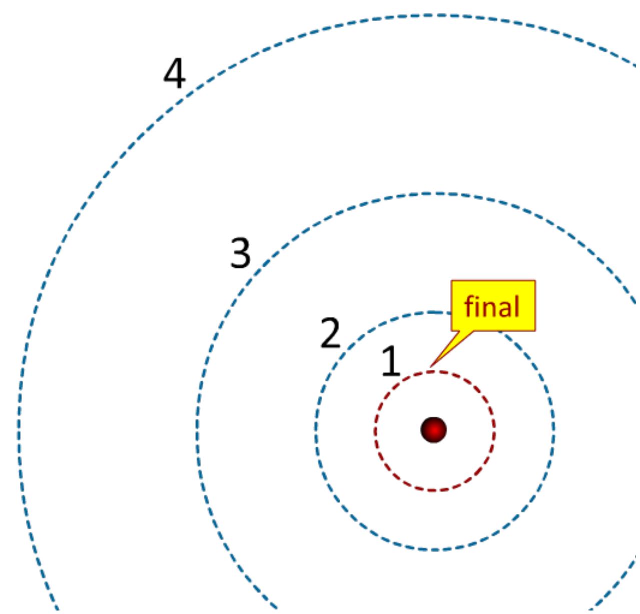
$$\lambda_{max} = \frac{4}{3} \times \frac{1}{R}$$

λ_{min} corresponds to largest possible energy change hence n_i $n_i \rightarrow \infty$

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right)$$

$$\lambda_{min} = \frac{1}{R}$$

Lyman series radiation lies in the ultra violet region



$$\boxed{\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

Balmer series

$n_f = 2$ and $n_i = 3, 4, 5, \ldots \infty$

λ_{max} corresponds to smallest energy change hence $n_i = 3$

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{4} - \frac{1}{9}\right)$$

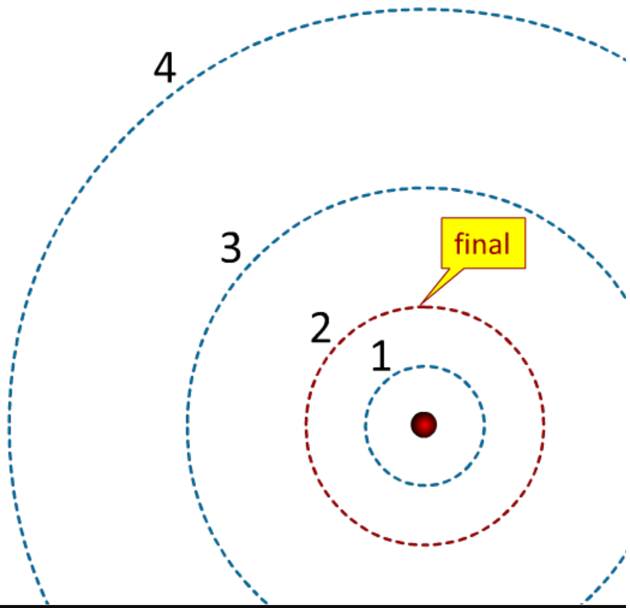
$$\lambda_{max} = \frac{36}{5}\frac{1}{R}$$

λ_{min} corresponds to largest possible energy change hence $n_i \rightarrow \infty$

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right)$$

$$\lambda_{min} = \frac{4}{R}$$

Balmer series radiation lies in the visible and near infra red region



$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Paschen series

$n_f = 3$ and $n_i = 4, 5, 6, \ldots \infty$

λ_{max} corresponds to smallest energy change hence $n_i = 4$

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{9} - \frac{1}{16}\right)$$

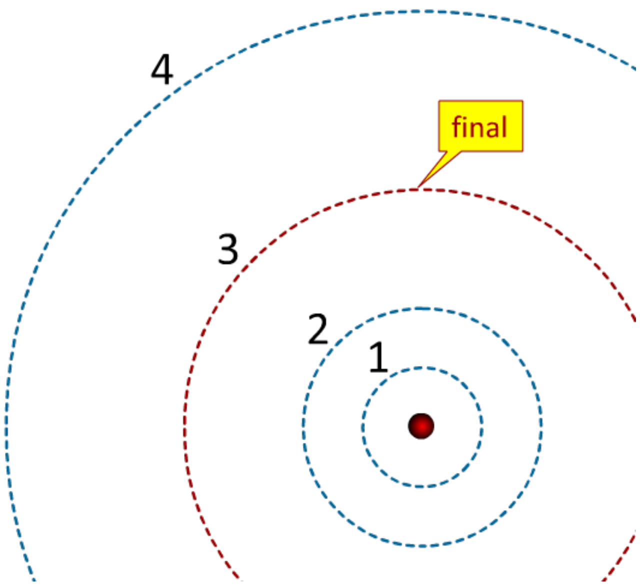
$$\lambda_{max} = \frac{144}{7}\frac{1}{R}$$

λ_{min} corresponds to largest possible energy change hence $n_i \rightarrow \infty$

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{3^2} - \frac{1}{\infty}\right)$$

$$\lambda_{min} = \frac{9}{R}$$

Paschen series radiation lies in the infra red region



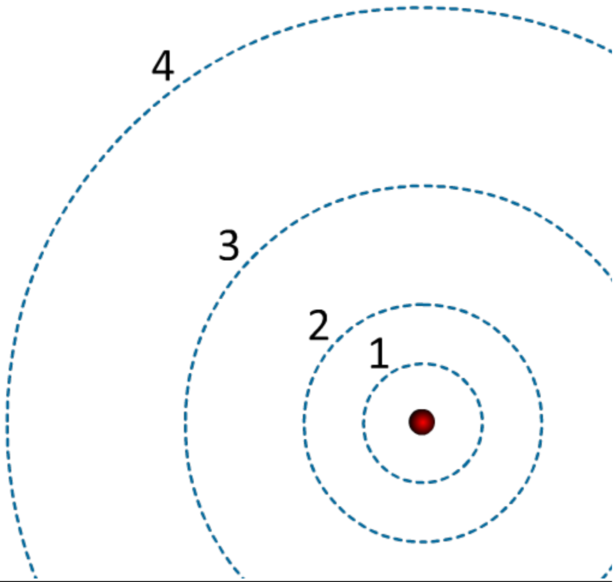
$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Brackett series

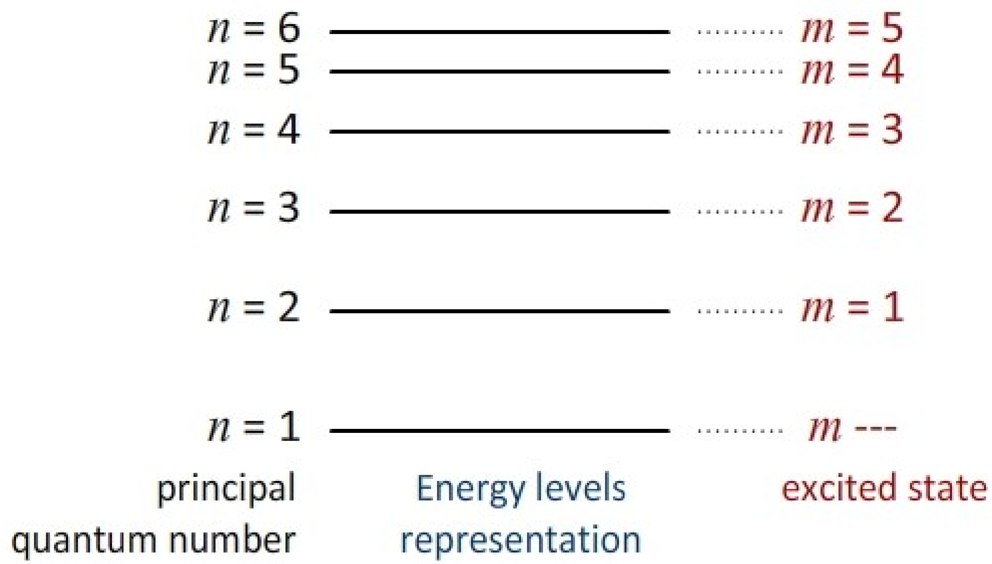
$n_f = 4$ and $n_i = 5, 6, 7, \ldots \infty$

Pfund series

$n_f = 5$ and $n_i = 6, 7, 8, \ldots \infty$



Some observations for transitions



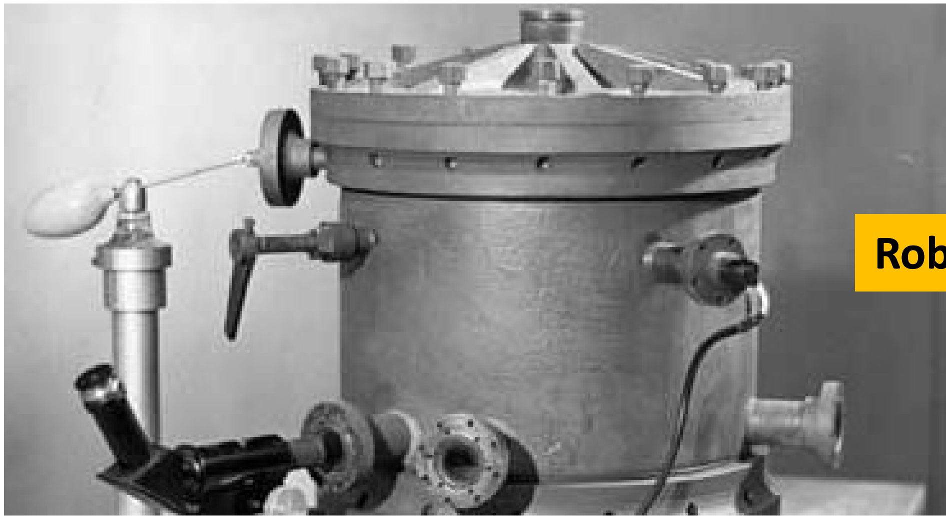
- Ground state is labeled as ZERO. The first excited state is labeled as 1, second excited state is labeled as 2, and so on.
- Energy levels become closer as we move to higher quantum numbers.
- Total number of possible transitions from the m^{th} excited state is given by

$$N_{\text{transitions}} = \frac{m(m+1)}{2}$$

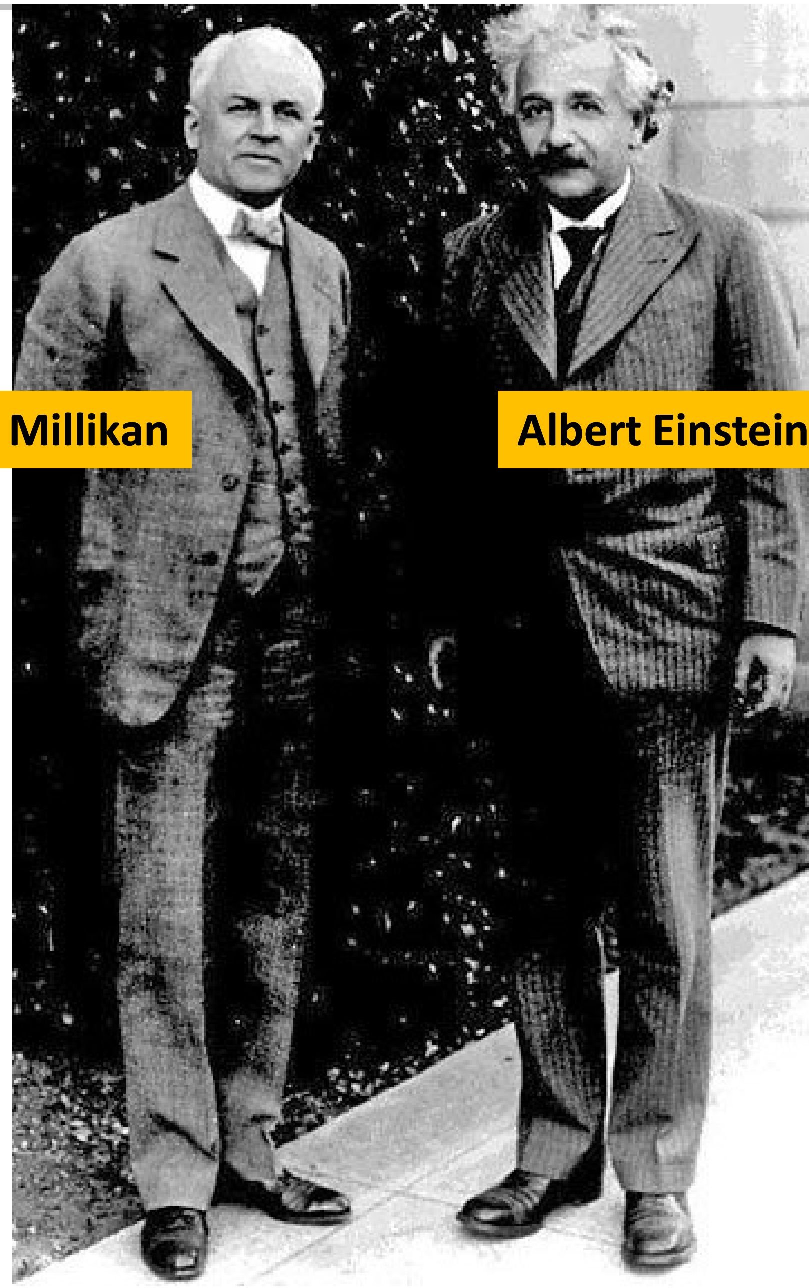
Limitations of Bohr's model

- Bohr's model is applicable only for hydrogen and hydrogen like atoms i.e. single electron atoms
- Bohr's model doesn't account for the elliptical nature of the orbits
- Bohr's model is unable to explain the relative intensities of the frequencies in the spectrum.
- Bohr's model cannot account for the fine splitting of spectral lines (Zeeman and Stark effects)

Determination of specific charge of electron



Original setup of the famous oil drop experiment

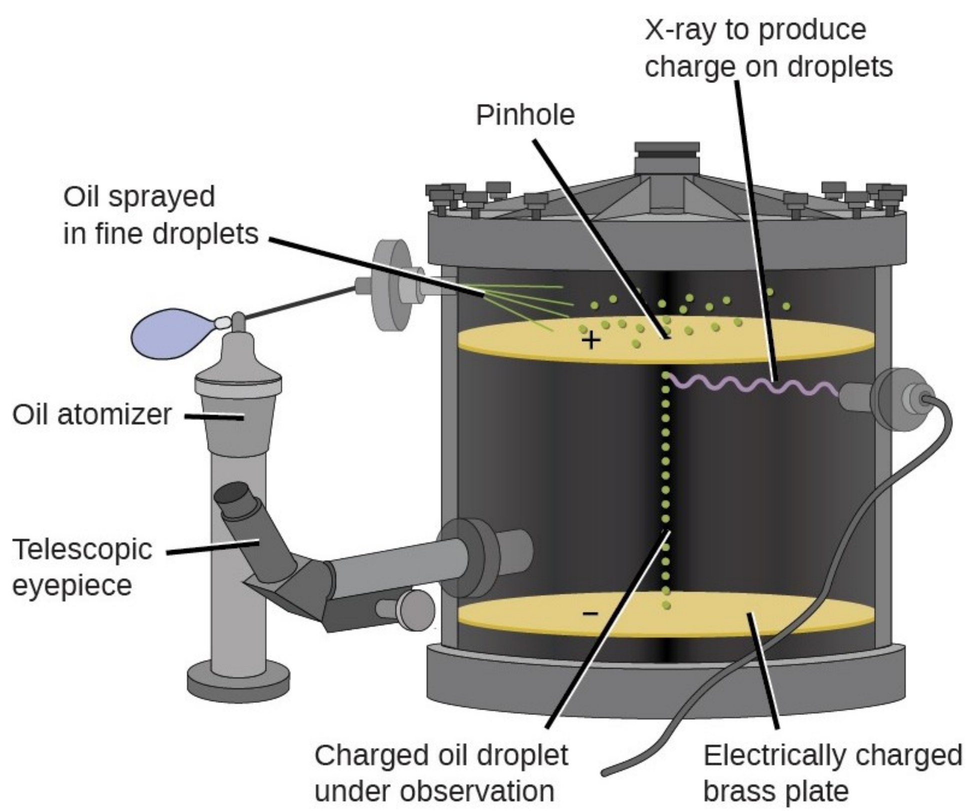


Experimental setup

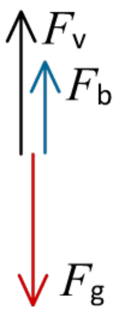
Millikan's oil drop experiment measured the charge of an electron by balancing gravitational and electric forces on tiny charged oil droplets. Charged droplets were sprayed into an electric field between two metal plates. Buoyant force, gravitational force and gravitational forces acting on the oil drop and the observed terminal velocity were used to calculate the radius of the oil drop.

By adjusting the electric field strength, droplets were suspended in equilibrium, allowing precise charge calculations.

Charge of the electron was found to be quantized.



R radius of oil drop v_g terminal velocity
d_1 density of air E electric field
d_2 density of oil drop
η coefficient of viscosity of air

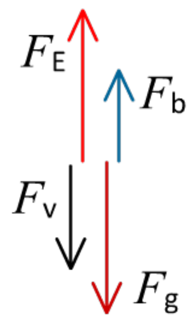


Oil drop attains terminal velocity under the influence of buoyant force, viscous and gravitational force.

$$\frac{4}{3}\pi R^3 d_1 g + 6\pi\eta R v_g = \frac{4}{3}\pi R^3 d_2 g$$

$$\frac{4}{3}\pi R^3 g\left(d_2 - d_1\right) = 6\pi\eta R v_g$$

$$R = \sqrt{\frac{9\pi\eta v_g}{2\pi g\left(d_2 - d_1\right)}} \quad \boxed{\text{i}}$$



In the presence of electric field, net force on the charged oil drop is again made zero (stationary)

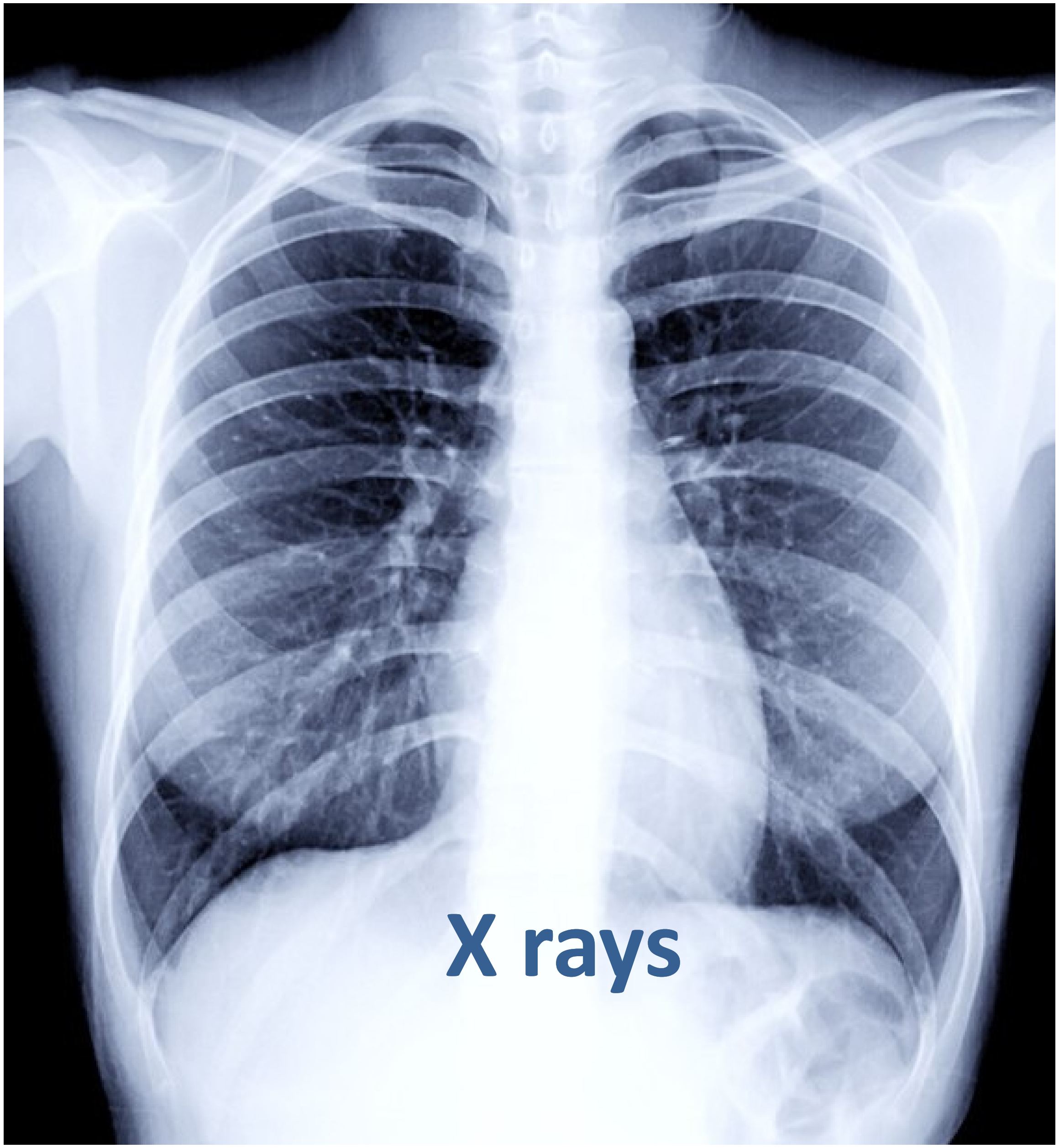
$$\frac{4}{3}\pi R^3 d_1 g + eE = \frac{4}{3}\pi R^3 d_2 g$$

Using R from eq (i)

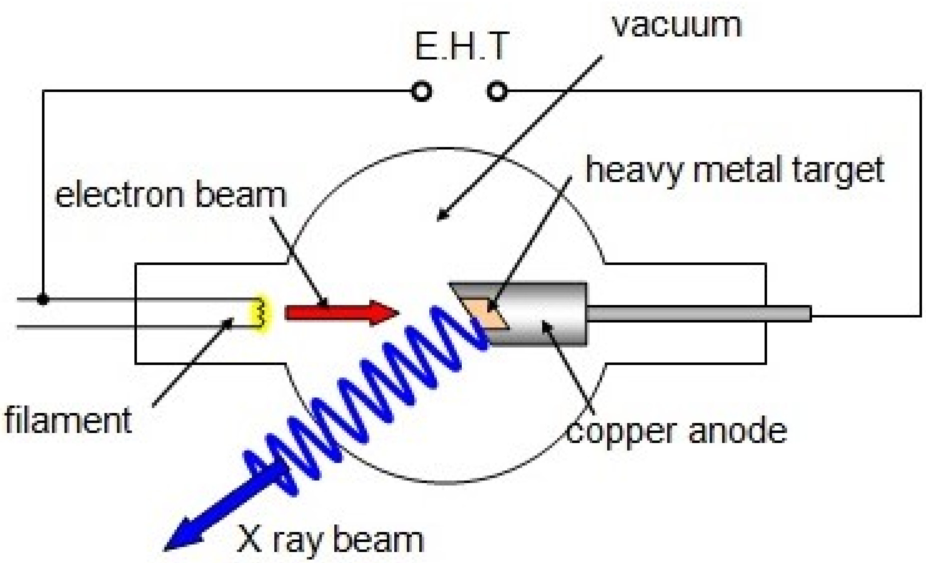
$$eE = \frac{4}{3}\pi R^3 g\left(d_2 - d_1\right)$$

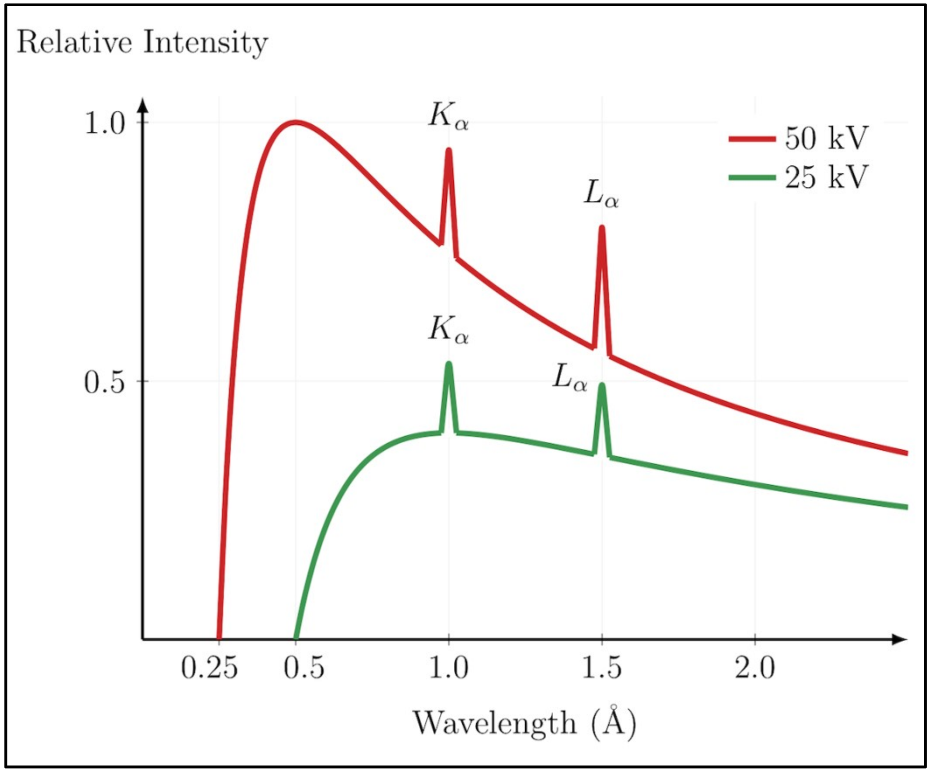
$$eE = \frac{4}{3}\pi\left(\frac{9\pi\eta v_g}{2\pi g\left(d_2 - d_1\right)}\right)^{3/2} g\left(d_2 - d_1\right)$$

$$e = \frac{4}{3}\pi\left(\frac{9\pi\eta v_g}{2\pi g\left(d_2 - d_1\right)}\right)^{3/2} \frac{g\left(d_2 - d_1\right)}{E}$$


X rays

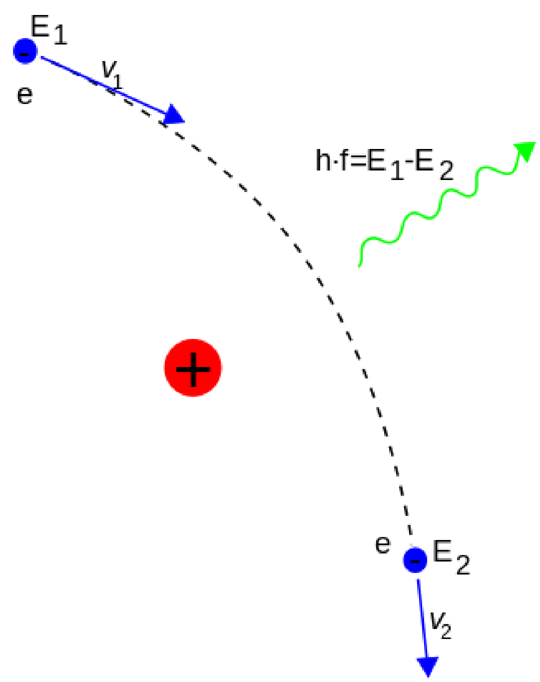
Production of X rays

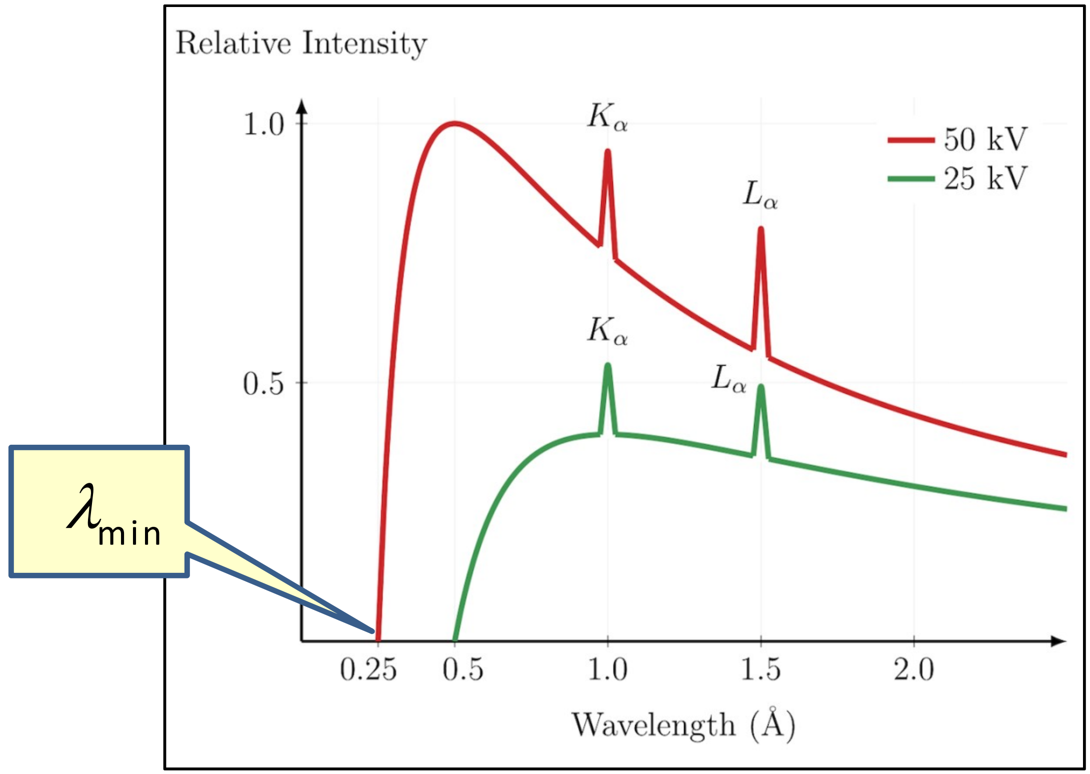




X-rays are produced in an X-ray tube where electrons are emitted from a heated filament (cathode) by thermionic emission and accelerated towards a metal target (anode) by a high voltage. When these high-speed electrons strike the anode, they suddenly decelerate, converting kinetic energy into X-ray photons giving rise to characteristic X-ray emission. The target is usually tungsten, chosen for its high melting point and atomic number. Only a small fraction of electron energy becomes X-rays. The remaining energy is dissipated as heat.

Continuous X-ray spectrum

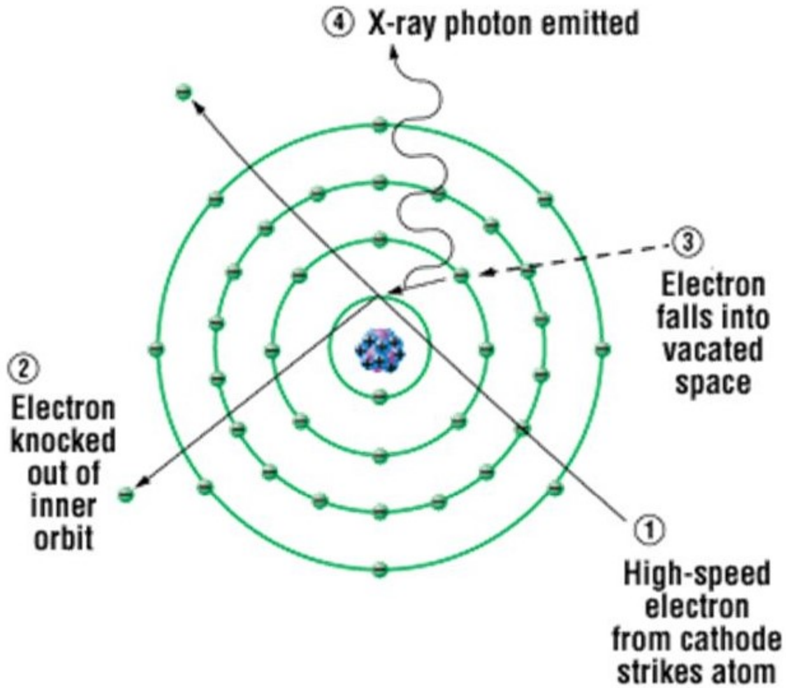


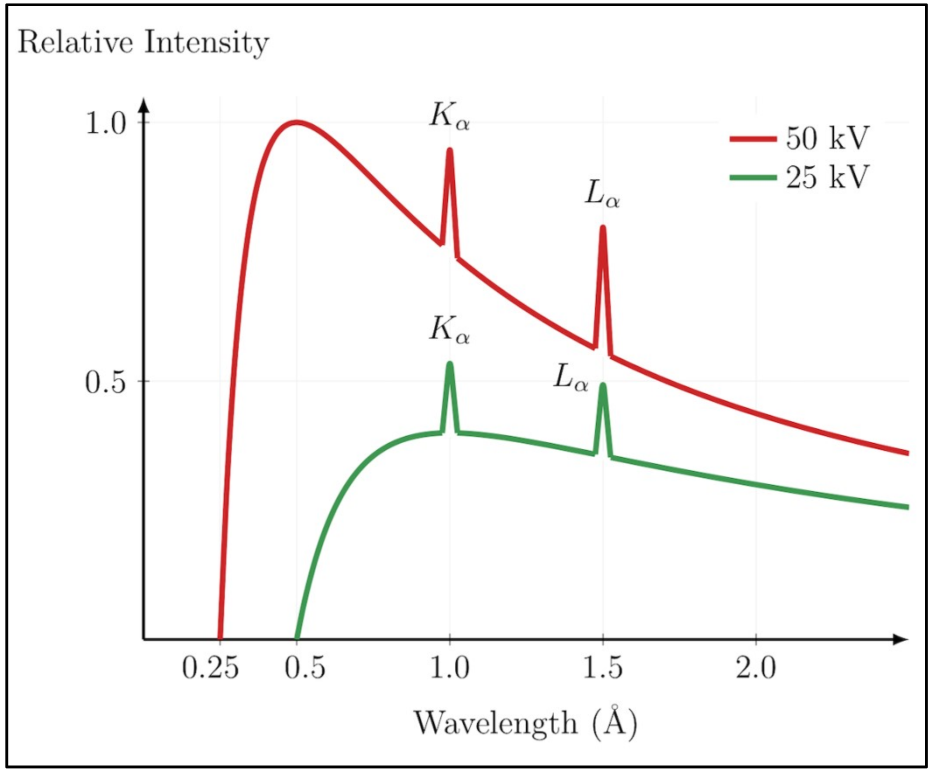


Electrons loose energy continuously as they are decelerated in the target. This gives to continuous emission of energy in the form of e.m. waves of continuously increasing wavelength.

If all the energy of the electron is lost in one collision then ALL the initial energy is lost in the form of X-ray of the maximum frequency (i.e. minimum wavelength)

Characteristic X-ray spectrum





When a high energy electron knocks off an electron from one of the inner shells of the atom, the resulting vacancy is filled by an electron from any higher energy level. Such a transition is accompanied by emission of radiation of specific wavelength depending on the difference of energy levels.

Moseley's law (empirical law)

$$\boxed{\sqrt{\nu} = a(Z - b)}$$

Total energy of the electron is given by

$$TE = -\frac{mZ^2e^4}{8\varepsilon_o^2 n^2 h^2}$$

Change in energy due to transition is

$$\Delta E = -\frac{mZ^2e^4}{8\varepsilon_o^2 h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$h\nu = \frac{mZ^2e^4}{8\varepsilon_o^2 h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

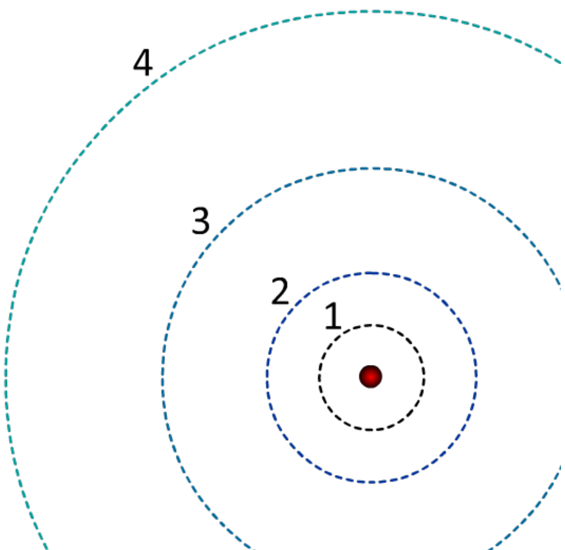
$$\nu = \frac{mZ^2e^4}{8\varepsilon_o^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\nu = \frac{me^4}{8\varepsilon_o^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)(Z - 1)^2$$

$$\sqrt{\nu} = a(Z - b)$$

a and b are constants

b = 1 for K_α line and 7.4 for L_α line



LASER

The acronym LASER stands for Light Amplification by Stimulated Emission of radiation.

In the case of laser light
(a) the wavelength of each packet is almost the same
(b) the average length of the packet of waves is much larger.

These properties lead to better phase correlation over a longer duration of time and result in reducing the divergence of a laser beam substantially.

There are low power lasers, with a power of 0.5 mW, called pencil lasers, which serve as pointers. There are also lasers of different power, suitable for delicate surgery of eye or glands in the stomach.
There are lasers which can cut or weld steel.